

**Example 3.2**  
 Let  $H = [0, 1]$  such that for any  $a, b \in [0, 1]$

$$a \circledast b = \begin{cases} [0, 0.3], & \text{if } b \neq 0 \text{ or } a = 0 = b \\ \{a\}, & \text{if } a \neq 0 \text{ and } b = 0 \end{cases}$$

Define a fuzzy set  $\mu$  in  $H$  by

$$\mu(a) = \begin{cases} l, & \text{if } a = 0 \\ k, & \text{if } a \neq 0 \end{cases} \text{ where } k < l.$$

**FUZZY HYPER GR-IDEAL of type 1**

|    | <i>x</i>          | <i>y</i>          | <i>x</i> $\circledast$ <i>y</i> | <i>u</i> $\in$ ( <i>x</i> $\circledast$ <i>y</i> ) | $\mu(u)$    | <i>S</i> = inf $\mu(u)$ | min{ <i>S</i> , $\mu(y)$ } | $\mu(x) \geq \min\{S, \mu(y)\}$ |
|----|-------------------|-------------------|---------------------------------|----------------------------------------------------|-------------|-------------------------|----------------------------|---------------------------------|
| 1) | 0                 | 0                 | [0, 0.3]                        | <i>u</i> $\in$ [0, 0.3]                            | <i>l, k</i> | <i>k</i>                | <i>k</i>                   | <i>l</i> > <i>k</i>             |
| 2) | <i>a</i> $\neq$ 0 | 0                 | { <i>a</i> }                    | <i>u</i> $\in$ { <i>a</i> }                        | <i>k</i>    | <i>k</i>                | <i>k</i>                   | <i>k</i> = <i>k</i>             |
| 3) | 0                 | <i>b</i> $\neq$ 0 | [0, 0.3]                        | <i>u</i> $\in$ [0, 0.3]                            | <i>l, k</i> | <i>k</i>                | <i>k</i>                   | <i>l</i> > <i>k</i>             |
| 4) | <i>a</i> $\neq$ 0 | <i>b</i> $\neq$ 0 | [0, 0.3]                        | <i>u</i> $\in$ [0, 0.3]                            | <i>l, k</i> | <i>k</i>                | <i>k</i>                   | <i>k</i> = <i>k</i>             |
| 5) |                   |                   |                                 |                                                    |             |                         |                            |                                 |
| 6) |                   |                   |                                 |                                                    |             |                         |                            |                                 |
| 7) |                   |                   |                                 |                                                    |             |                         |                            |                                 |
| 8) |                   |                   |                                 |                                                    |             |                         |                            |                                 |

**Example 3.3**  
 Consider the hyper GR-algebra  $H$  in Example ??. Define a fuzzy set  $\mu$  in  $H$  by

$$\mu(x) = \begin{cases} 1, & \text{if } x = 0, \\ 0.3 + x, & \text{if } x \in (0, 0.3], \\ 0.7, & \text{if } x \in (0.3, 1]. \end{cases}$$

|    | $x$        | $z$        | $A = x \otimes z$ | $\min\{\inf \mu(v), \mu(z)\}$ | $\mu(x) \geq \min\{S, \mu(z)\}$ |
|----|------------|------------|-------------------|-------------------------------|---------------------------------|
| 1) | $x \neq 0$ | 0          | $\{x\}$           | 0.3                           | $\mu(x) \geq 0.3$               |
| 2) | 0          | 0          | $[0, 0.3]$        | 0.3                           | $1 > 0.3$                       |
| 3) | $x \neq 0$ | $z \neq 0$ | $[0, 0.3]$        | 0.3                           | $\mu(x) \geq 0.3$               |
| 4) | 0          | $z \neq 0$ | $[0, 0.3]$        | 0.3                           | $1 > 0.3$                       |

It follows from the tables that  $\mu$  is a fuzzy GR-ideal of type 1.

Example 4.2

| IFHGR2 and IFHGR3 |            |            |              |                                          |                       |                                      |                                             |                          |                                            |
|-------------------|------------|------------|--------------|------------------------------------------|-----------------------|--------------------------------------|---------------------------------------------|--------------------------|--------------------------------------------|
|                   | $x$        | $y$        | $x \oplus y$ | $S = \inf_{u \in (x \oplus y)} \mu_A(u)$ | $\min\{S, \mu_A(y)\}$ | $\mu_A(x) \geq \min\{S, \mu_A(y)\}?$ | $T = \sup_{u \in (x \oplus y)} \gamma_A(u)$ | $\max\{T, \gamma_A(y)\}$ | $\gamma_A(x) \leq \max\{T, \gamma_A(y)\}?$ |
| 1)                | $x \neq 0$ | 0          | $\{x\}$      | 0.3                                      | 0.3                   | $\mu_A(x) \geq 0.3$                  | 0.7                                         | 0.7                      | $\gamma_A(x) \leq 0.7$                     |
| 2)                | 0          | 0          | $[0, 0.3]$   | 0.3                                      | 0.3                   | $1 > 0.3$                            | 0.7                                         | 0.7                      | $0 < 0.7$                                  |
| 3)                | $x \neq 0$ | $y \neq 0$ | $[0, 0.3]$   | 0.3                                      | 0.3                   | $\mu(x) \geq 0.3$                    | 0.7                                         | 0.7                      | $\gamma_A(x) \leq 0.7$                     |
| 4)                | 0          | $y \neq 0$ | $[0, 0.3]$   | 0.3                                      | 0.3                   | $1 > 0.3$                            | 0.7                                         | 0.7                      | $0 < 0.7$                                  |

Note that IFGR1 is clearly satisfied.  
Thus,  $A = (\mu_A, \gamma_A)$  is an intuitionistic fuzzy hyper GR-deal in  $H$ .